The modal nature of colors in higher-order model-checking

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Abstract. A careful investigation of the work by Naoki Kobayashi and Luke Ong reveals that priorities (or colors) behave in essentially the same way as the exponential modality of linear logic. From this follows a purely proof-theoretic reconstruction of their type system, based on an indexed and infinitary variant of tensorial logic. In this talk, we will describe the basic properties of the color modalities, and how to translate a minor variant of Kobayashi and Ong's type system into tensorial logic with colors.

We would like to speak for about 30 minutes.

A brief overview of the talk

Our starting point in this talk will be provided by the recent but already influential work by Kobayashi and Ong [1] where the two authors associate to every alternating parity tree automaton $\mathcal{A}$ on a given signature Σ an intersection type system $KO(\mathcal{A})$ with colors. Kobayashi and Ong then establish a fundamental correspondence theorem, which states that given a higher-order recursion scheme $\mathcal{G}$ on the same signature Σ, the existence of a winning run-tree of the automaton $\mathcal{A}$ on the value-tree $[\![\mathcal{G}]\!]$ generated by $\mathcal{G}$ is equivalent to the existence of a winning strategy for Eve in a parity game $\boldsymbol{Adamic}(\mathcal{G}, \mathcal{A})$ formulated on the type system $KO(\mathcal{A})$. We will introduce a variant $\boldsymbol{Edenic}(\mathcal{G}, \mathcal{A})$ of the original parity game $\boldsymbol{Adamic}(\mathcal{G}, \mathcal{A})$ and a variant $KO_{fix}(\mathcal{G}, \mathcal{A})$ of the original type system $KO(\mathcal{A})$ and establish a one-to-one correspondence between

- the winning strategies σ of Eve played in the parity game $\boldsymbol{Edenic}(\mathcal{G}, \mathcal{A})$,
- the winning derivation trees π constructed in the type system $KO_{fix}(\mathcal{G}, \mathcal{A})$ and with conclusion the type judgement

$$S : \boxdot_{\Omega(q_0)} q_0 :: \bot \quad \vdash \quad S : q_0 :: \bot. \tag{1}$$

where the non-terminal S denotes the "axiom" of the higher-order recursion scheme $\mathcal{G}$ while the box $\boxdot_{\Omega(q_0)}$ is simply a tag informing the type system $KO_{fix}(\mathcal{G}, \mathcal{A})$ that the initial state q_0 has colour $\Omega(q_0)$ in the alternating parity tree automaton $\mathcal{A}$. An important point is that the winning condition on derivation trees of $KO_{fix}(\mathcal{G}, \mathcal{A})$ is extremely simple: a derivation tree π is declared *winning* precisely when all its *infinite* branches b are winning — in the obvious sense that the maximal colour m_b encountered an *infinite* number of times in the branch b is even, rather than odd. By composing this correspondence theorem with the theorem established by Kobayashi and Ong [1], we obtain that

Correspondence theorem. *The alternating parity tree automaton $\mathcal{A}$ has a winning run-tree over the value-tree $[\![\mathcal{G}]\!]$ precisely when there exists a winning derivation tree with conclusion (1) in the type system $KO_{fix}(\mathcal{G}, \mathcal{A})$.*

This result is important because it reveals the truly proof-theoretic nature of the parity game ***Adamic***$(\mathcal{G}, \mathcal{A})$ originally designed by Kobayashi and Ong in [1]. Indeed, the winning condition is reformulated here as an elementary winning condition on an infinite derivation trees. This observation leads us to the second aspect of our presentation, which will be to elaborate a translation of the type system $KO_{fix}(\mathcal{G}, \mathcal{A})$ designed by Kobayashi and Ong into a refinement of linear logic called *tensorial logic* and developed by the second author in a series of recent articles [2–4]. The translation into tensorial logic relies on the unexpected discovery that the colors appearing in the type system $KO_{fix}(\mathcal{G}, \mathcal{A})$ behave in essentially the same way as the exponential modality ! of linear logic, or as the necessity modality $\Box$ of the modal logic S_4. In order to translate $KO_{fix}(\mathcal{G}, \mathcal{A})$ into tensorial logic, we thus extend the logic with a specific modality noted $\Box_m$ for each colour $m \in \mathbb{N}$, and equipped with the structure of a parametric monoidal comonad in the sense of [4]. This means proof-theoretically that the following sequents are canonically provable in the logic:

$$\begin{array}{ccc} \Box_0\, A & \vdash & A \\ \Box_{max(m_1,m_2)}\, A & \vdash & \Box_{m_1}\, \Box_{m_2}\, A \\ \Box_m A \otimes \Box_m B & \vdash & \Box_m\, (\, A \otimes B\,) \end{array}$$

The resulting logic defines what we call *tensorial logic with colors.* Every derivation tree π of $KO_{fix}(\mathcal{G}, \mathcal{A})$ is then translated into a derivation tree $[\pi]$ of this logic, moreover extended with intersection types. We establish the following

Embedding theorem. *A derivation tree π of $KO_{fix}(\mathcal{G}, \mathcal{A})$ with conclusion (1) is winning if and only if its translation $[\pi]$ of conclusion*

$$\Gamma \vdash [\mathcal{G}] \,:\, q_0 \,::\, \bot \tag{2}$$

defines a winning derivation tree of tensorial logic with colors.

Here, the infinite $\beta\eta$-long normal form $[\mathcal{G}]$ is an infinite λ-term deduced by a series of canonical and elementary transformations on the original higher-order recursion scheme $\mathcal{G}$. On the other hand, the context Γ is a refinement of the simple types associated to the constructors of the signature Σ. This refinement reflects the interaction between the higher-order recursion scheme $\mathcal{G}$ and the alternating parity tree automaton $\mathcal{A}$. Recall that in the traditional interpretation of trees inherited from the work on polymorphism and parametricity by Girard and Reynolds, a tree of signature

$$\Sigma \quad = \quad \{ a : 2 \, , b : 1 \, , c : 0 \}$$

is interpreted as a λ-term of type

$$(o \to o \to o) \to (o \to o) \to o \to o.$$

In the case of this signature the context Γ would refine the types

$$a : o \to o \to o \qquad b : o \to o \qquad c : o$$

of the three constructors a, b, c of the signature. An important methodological point about the translation is that the definition of tensorial logic is independent of $\mathcal{G}$ and $\mathcal{A}$, in contrast to the approach developed by Kobayashi and Ong where the definition of the type system $KO(\mathcal{A})$ depends on $\mathcal{A}$. Hence, one recovers in this way the traditional interpretation of trees inherited from the work on polymorphism and parametricity by Girard and Reynolds.

Details of the construction will be given during the talk.

A series of related works

Propelled by the seminal result by Ong [5], higher-order model-checking has become a very active topic in the recent years. In particular, much work has been devoted to reestablish the original decidability result by Luke Ong. Besides the type-theoretic approach developed by Kobayashi and Ong [6, 1], Hague, Murawski, Ong, Serre [7] developed an automata-theoretic approach based on the translation of the higher-order recursion scheme $\mathcal{G}$ into a collapsible pushdown automaton (CPDA), which led the four authors to yet another proof of Luke Ong's decidability theorem. An important and clarifying connection was then made by Salvati and Walukiewicz between this translation of higher-order recursion schemes into CPDAs and the traditional evaluation mechanism of the environment Krivine machine [8]. Following this discovery, Salvati and Walukiewicz are currently developing a semantic approach to higher-order model checking, based on the interpretation of the Krivine environment machine in finite models of the λ-calculus with fixpoint operators, see [9, 10] for details.

Besides its application to the concrete implementation of higher-order model checkers [11, 12], the type-theoretic approach to higher-model checking initiated by Kobayashi and Ong [6, 1] has attracted a lot of theoretical interest in the community. In particular, Haddad developed in his PhD thesis [13] an automata-theoretic reformulation of the Kobayashi-Ong type system $KO(\mathcal{A})$ and companion parity game $\boldsymbol{Adamic}(\mathcal{G}, \mathcal{A})$. Thanks to this reformulation, Haddad was able to give a constructive proof of the decidability of the selection problem for monadic second-order logic, see [14, 13] for details.

The idea of connecting linear logic to automata theory is a longstanding dream which has been nurtured by a number of important contributions. Among them, we would like to mention the thorough categorical and proof-theoretic study by Santocanale [15–17] of the connections between circular proofs, μ-bicomplete categories, and the modal μ-calculus. Another source of inspiration has been the work by Baelde [18] on a multiplicative additive linear logic $\mu MALL$ extended with a dual pair of induction (μ) and coinduction (ν) operators on the formulas. Finally, let us also mention the recent work by Terui [19] who develops a semantic and type-theoretic approach based on linear logic, intersection types and automata theory in order to characterize the complexity of evaluation to the booleans in the simply-typed λ-calculus. See the related work on intersection types and complexity by de Carvalho [20]. We will also mention in our conclusion the promising connection of this work with Bucciarelli and Ehrhard's *indexed linear logic* introduced in [21, 22].

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